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Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (341) 624-98-75

A JOINT ESTIMATION OF THE PRODUCTION FUNCTION AND THE DEPRECIATION RATE OF THE CAPITAL STOCK. A DISAGGREGATED ANALYSIS.

Ignacio Mauleón and Marta Risueño*

Abstract

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Keywords: Depreciation rate, capital stock, random coefficients, pooled data.

* Mauleón, Universidad de Salamanca; Risueño, Departamento de Economía, Universidad Carlos III de Madrid. E-mail: Risueño@eco.uc3m.es.

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In this article we estimate the depreciation rate of the capital stock jointly with the parameters of a production function. We use data on 81 Spanish manufacturing sectors for the period 1978-1992. The methodology used allows us to obtain consistent capital stock series using gross investment data and an initial capital stock.

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1. INTRODUCTION

The aim of this article is the joint estimation of the depreciation rate of the capital stock and the parameters of a production function. The study is based on sectorial data of the Spanish industry taken from the Encuesta Industrial¹, for the period 1978-1992.

The article is based on the research line first started by Nadiri & Prucha (1993), who estimated jointly the parameters of a cost function and the depreciation rate of capital using time series data. More recently Denia et al (1996), use Spanish data to estimate a production function jointly with the depreciation rate of capital, and Nadiri & Prucha (1996) estimate factor demands and calculate the depreciation rate of capital in different periods.

The study makes contributions to the existing literature in various respects. On the one hand it uses disaggregated data which allows the study of the behavior of the depreciation rate in various sectors. The data consists of time series for different productive sectors, where the size of the time series, T , and the number of sectors, N , are large. That is, the asymptotic properties of the estimators and inference tests are justified for $T \rightarrow \infty$ and $N \rightarrow \infty$. This type of panels is known as data fields (Quah, 1990) in order to distinguish them from the typical panels used in microeconometrics where T is generally small. The parameters of the production function do not tend to be stable among the productive sectors or among different firms, as has been observed by other authors (i.e. Mairesse & Griliches, 1990). When the parameters among sectors are not constant, the estimation of the mean value, using the usual estimation techniques for microeconomic panels, tends to be invalid. Therefore inferences on the mean value of the parameters using pooled data and estimations for each sector are provided. On the other hand, this article studies the problem posed by the introduction of technological change in the usual manner, via a deterministic

¹An industrial questionnaire.

trend. It is shown that there is an identification problem between the coefficients of the deterministic trend and the parameter of the depreciation rate of capital. Thus we suggest alternative ways of incorporating technical progress.

The rest of the article is organized as follows. Section 2 discusses the econometric model and estimation techniques. Section 3 presents the data and alternative specifications of the model, analyzing the introduction of technical progress. Section 4 gathers the estimation results of a random coefficients model where the coefficients of each sector are estimated from the time series. Section 5 presents the results using pooled data, imposing the same coefficients for all sectors or group of sectors. Section 6 compares the depreciation rates and capital stock series estimated in the present study with those obtained in another article using the same sample. Finally section 7 gathers the main conclusions and final comments.

2. ECONOMETRIC MODEL AND ESTIMATION TECHNIQUES

Suppose we have data $\{Y_{it}, I_{it}, L_{it}, Z_{it}, i = 1, \dots, N \text{ and } t = 1, \dots, T\}$, of different variables over N productive sectors in T periods of time:

Y_{it} = Production of sector i in period t .

I_{it} = Gross Investment of sector i in period t .

L_{it} = Labor input of sector i in period t .

Z_{it} = Other variables that explain the technology of sector i in period t . (such as variables that explain technical progress, etc.).

Each variable is an aggregate of the firms that make up the sector. Therefore, Y_{it} , I_{it} , L_{it} and Z_{it} are, for each i , an economic time series. The characterization of the variables depends on the specification chosen, as will be discussed in the next section.

There are N productive sectors and T periods of time. Both the number of sectors and the number of time periods, is large (in the statistical sense). Therefore, inferences on the basis that $T \rightarrow \infty$ and $N \rightarrow \infty$ are justified.

The capital stock of manufacturing sector i in period t is assumed to be generated by

$$\begin{aligned} K_{it}(\delta_i) &= I_{it} + (1 - \delta_i)K_{it-1}(\delta_i) \\ &= \sum_{s=0}^{t-1} (1 - \delta_i)^s I_{it-s} + (1 - \delta_i)^t K_{i0} \\ &= K(I_{i1}, I_{i2}, \dots, I_{it}, K_{i0}, \delta_i), \end{aligned} \quad (1)$$

where K_{i0} is taken as given. Usually equation (1) is used to generate a capital stock series, giving an arbitrary value to δ_i . Nevertheless, if we accept that the depreciation rate is unknown, the capital stock series, $K_{it}(\delta_i)$, cannot be calculated. This article proposes the estimation of the depreciation rate, δ_i , jointly with the parameters of a production function.

The econometric model that characterizes the technology for each sector is represented by the conditional moments restriction:

$$E\{Y_{it}|I_{it}, I_{it-1}, \dots, I_{i1}, K_{i0}, L_{it}, Z_{it}\} = f_i(K_{it}(\delta_i), L_{it}, Z_{it}, \theta^{(i)}), \quad (2)$$

where $f_i(\cdot)$ is the production function of sector i and $\theta^{(i)}$ is a vector of parameters. Note that, although f_i is linear on $K_{it}(\delta_i)$, L_{it} , Z_{it} and $\theta^{(i)}$, it will not be linear in δ_i .

The estimation of δ_i and $\theta^{(i)}$ can be carried out by non-linear least squares. However, a difficulty arises, since (1) has different arguments for each t . In order to express (1) in the usual way, where the same explanatory variables are used in each equation, we follow Prucha (1995), and redefine the investment and initial capital stock variables. Equation (1) can be expressed in the following alternative manner:

$$K_{it}(\delta_i) = \sum_{j=1}^t (1 - \delta_i)^{t-j} I_{ij} + (1 - \delta_i)^t K_{i0}. \quad (3)$$

If we define $I_{it}^{(j)} = I_{ij} D_t^{(j)}$ where $D_t^{(j)} = \begin{cases} 1 & \text{for } t \geq j \\ 0 & \text{for } t < j \end{cases}$,

we have T explanatory variables:

$$I_{it}^{(1)}, \dots, I_{it}^{(T)}, \quad i = 1, \dots, N, \quad t = 1, \dots, T. \quad (4)$$

so that we can rewrite

$$K_{it}(\delta_i) \equiv K(I_{it}^{(1)}, \dots, I_{it}^{(T)}, \delta_i, K_{i0}) = \sum_{s=1}^T (1 - \delta_i)^{t-s} I_{it}^{(s)} + (1 - \delta_i)^t K_{i0}, \quad (5)$$

with this new specification each regression equation has the same list of arguments in each period of time and a standard econometric package can be used.

We choose a Cobb-Douglas functional form because of its simplicity, and because it is the least non-linear specification. Likewise, this functional form is a first order approximation to many technologies. Under this specification the production function may be written as:

$$f_i(K_{it}(\delta_i), L_{it}, Z_{it}, \theta^{(i)}) = A_i [K_{it}(\delta_i)]^{\beta_{Ki}} L_{it}^{\beta_{Li}} g_i(Z_{it}, \gamma_i), \quad (6)$$

where $\theta^{(i)} = (A_i, \beta_{Ki}, \beta_{Li}, \gamma_i)'$ and $g_i(\cdot)$ is a function that takes account of other effects which are important in the specification of the technology of the sector throughout time, such as technological progress, and structural changes. While the parameters β_{Ki} and β_{Li} have a precise interpretation in terms of elasticities, the way in which $g_i(\cdot)$ is modelled tends to be arbitrary, in general following the criteria of computational simplicity. It is useful to adopt the following expression:

$$g_i(Z_{it}, \gamma_i) = \exp \{ Z_{it}' \gamma_i \},$$

so that under regularity conditions of the process that generate the data, we may write:

$$Y_{it} = B_i [K_{it}(\delta_i)]^{\beta_{Ki}} L_{it}^{\beta_{Li}} \exp \{ Z_{it}' \gamma_i + \varepsilon_{it} \},$$

where

$$\begin{aligned} E\{e^{\varepsilon_{it}} | I_{it}, I_{it-1}, \dots, I_{i1}, K_{i0}, L_{it}, Z_{it}\} &= A_i/B_i \\ y \ E\{\varepsilon_{it} | I_{it}, I_{it-1}, \dots, I_{i1}, K_{i0}, L_{it}, Z_{it}\} &= 0. \end{aligned}$$

Therefore the production function may be expressed by means of the regression equation:

$$\ln Y_{it} = \alpha_i + \beta_{Ki} \ln K_{it}(\delta_i) + \beta_{Li} \ln L_{it} + \gamma_i' Z_{it} + \varepsilon_{it}, \quad (7)$$

which is linear in all parameters but δ_i .

If our aim is to estimate the mean values of the coefficients, it is useful to assume the following specification,

$$\begin{aligned}\alpha_i &= \alpha + \eta_{1i} \\ \beta_{Ki} &= \beta_K + \eta_{2i} \\ \beta_{Li} &= \beta_L + \eta_{3i} \\ \gamma_i &= \gamma + \eta_{4i} \\ \delta_i &= \delta + \eta_{5i}\end{aligned}\tag{8}$$

where η_i , is a vector of independent and identically distributed with respect to i , random variables with $E(\eta_{ki} | K_{it}(\delta_i), L_{it}, Z_{it}) = 0$ and $E(\eta_i \eta_i') = V, \eta_i = (\eta_{i1}, \dots, \eta_{i5})$. In a more compact form, we can express $\phi^{(i)} = (\alpha_i, \beta_{Ki}, \beta_{Li}, \gamma_i, \delta_i')'$, $\phi = (\alpha, \beta_K, \beta_L, \gamma', \delta)'$ so that:

$$\phi^{(i)} = \phi + \eta_i.$$

Let's assume that $\delta_i = \delta$ (that is, $\sigma_\delta^2 = 0$). Then we can write:

$$\ln Y_{it} = \alpha + \beta_K \ln K_{it}(\delta) + \beta_L \ln L_{it} + \gamma' Z_{it} + v_{it},\tag{9}$$

where

$$v_{it} = \varepsilon_{it} + \eta_{1i} + \eta_{2i} \ln K_{it}(\delta) + \eta_{3i} \ln L_{it} + \eta_{5i}' Z_{it}.\tag{10}$$

and, therefore, the errors v_{it} have zero conditional mean and are conditionally heteroscedastic. Due to the temporary dependence in $\ln K_{it}(\delta)$ and $\ln L_{it}$, the v_{it} will be autocorrelated even if the ε_{it} are not. If $\ln K_{it}(\delta)$ or $\ln L_{it}$ are non-stationary, v_{it} will not be stationary either and equation (10) will not be a co-integration relation. The usual procedure of introducing lagged values of the dependent and explanatory variables in order to reduce the correlation produces inconsistent estimators as demonstrated by Robertson & Symons (1993) and Pesaran & Smith (1995). If $\ln K_{it}(\delta_i)$ and $\ln L_{it}$ are stationary and are not correlated with ε_{it} and η_i , the non-linear least squares (NLLS) estimator of the parameters ϕ in equation (9), using the NT observations, will be consistent, provided the assumption of mean independence between

η_{ik} and the explanatory variables can be maintained. The estimator may be express as:

$$\check{\phi} = \arg \min_{\phi} \sum_{i=1}^N \sum_{t=1}^T [\ln Y_{it} - \alpha - \beta \ln K_{it}(\delta) - \beta_L \ln L_{it} - \gamma' Z_{it}]^2.$$

The estimator is consistent for ϕ . Nevertheless, if $\ln K_{it}(\delta_i)$ or $\ln L_{it}$ have a unit root, the errors v_{it} will also have a unit root and the NLLS estimator, using all the observations, will be inconsistent. The variance-covariance matrix of this estimator may be approximated by:

$$\hat{\Omega} = \left[\sum_{i=1}^N \sum_{t=1}^T \check{x}_{it} \check{x}_{it}' \right]^{-1} \sum_{i=1}^N \sum_{t=1}^T \sum_{s=1}^T \check{x}_{it} \check{x}_{it}' \check{v}_{it} \check{v}_{is}' \left[\sum_{i=1}^N \sum_{t=1}^T \check{x}_{it} \check{x}_{it}' \right]^{-1} \quad (11)$$

as suggested by Arellano & Bover (1990) in a linear regression context, where

$$\check{x}_{it} = \partial v_{it}(\check{\phi}) / \partial \phi, \check{v}_{it} = v_{it}(\check{\phi}) \text{ y } v_{it}(\phi) = \ln Y_{it} - \alpha - \beta \ln K_{it}(\delta) - \beta_L \ln L_{it} - \gamma' Z_{it}.$$

An approximation to the true asymptotic variance of the errors of the model could be obtained by nonparametric methods as suggested by Newey & West (1987), among others. The small size of the time series prevents the implementation of this procedure. For this reason, we have used the usual method in panels where the time series is short. We have estimated the variance-covariance matrix, in a context where heteroskedasticity and autocorrelation of unknown functional form may be present.

The situation is more complicated when δ_i varies among sectors (that is, $\sigma_{\delta}^2 \neq 0$). From equation (9), we obtain,

$$v_{it} = \varepsilon_{it} + \eta_{1i} + \eta_{2i} \ln K_{it}(\delta) + \eta_{3i} \ln L_{it} + \eta_{4i}' Z_{it} + \beta_K [\ln K_{it}(\delta_i) - \ln K_{it}(\delta)].$$

In this case,

$$\begin{aligned} & E\{v_{it} | I_{it}, I_{it-1}, \dots, I_{i1}, K_{i0}, L_{it}, Z_{it}\} \\ &= \beta_K E\{\ln K_{it}(\delta_i) - \ln K_{it}(\delta) \mid I_{it}, I_{it-1}, \dots, I_{i1}, K_{i0}, L_{it}, Z_{it}\}. \end{aligned} \quad (12)$$

If the conditional expectation is zero and $\ln K_{it}(\delta_i)$ and $\ln L_{it}$ are stationary, δ , β_K , β_L and γ can be consistently estimated by NLLS. Nevertheless, it seems difficult to

justify that the conditional expectation is not going to depend on the explanatory variables, in which case the NLLS estimators are inconsistent even when $\ln K_{it}(\delta_i)$ and $\ln L_{it}$ are stationary. The non-linearity of the parameters, makes the specification of the model incorrect whenever δ_i is taken as fixed. However, the NLLS estimators calculated for each sector are consistent, -even super consistent in the non-stationary case-, which allows the estimation of ϕ from these estimators, as detailed below.

An alternative way of proceeding is to estimate the parameters $\phi^{(i)}$ with the T time series observations for each sector i by NLLS. Let $\hat{\phi}^{(i)} = (\hat{\alpha}_i, \hat{\beta}_{Ki}, \hat{\beta}_{Li}, \hat{\gamma}_i, \hat{\delta}_i)'$ the NLLS estimator of $\phi^{(i)}$. That is:

$$\hat{\phi}^{(i)} = \arg \min_{\phi} \sum_{t=1}^T [\ln Y_{it} - \alpha_i - \beta_i \ln K_{it}(\delta_i) - \beta_{Li} \ln L_{it} - \gamma_i' Z_{it}]^2, i = 1, \dots, 81.$$

These estimators are consistent when $\ln K_{it}(\delta_i)$ and $\ln L_{it}$ are stationary and when they are not. From this estimator, $\phi = (\alpha, \beta_K, \beta_L, \gamma', \delta)'$ is estimated consistently by:

$$\tilde{\phi} = \frac{1}{N} \sum_{i=1}^N \hat{\phi}^{(i)}$$

It follows that:

$$\begin{aligned} \sqrt{N}(\tilde{\phi} - \phi) &= \frac{1}{\sqrt{N}} \sum_{i=1}^N (\phi^{(i)} - \phi) + \frac{1}{\sqrt{N}} \sum_{i=1}^N (\hat{\phi}^{(i)} - \phi^{(i)}) \\ &= \frac{1}{\sqrt{N}} \sum_{i=1}^N (\phi^{(i)} - \phi) + O_p(\sqrt{N}T^{-\kappa}), \end{aligned}$$

where ϕ is defined as above; $\kappa = 1/2$ when $\ln K_{it}(\delta_i)$ and $\ln L_{it}$ are stationary with weak memory, $1/2 < \kappa < 1$ when $\ln K_{it}(\delta_i)$ or $\ln L_{it}$ are stationary with long memory, and $\kappa = 1$ when $\ln K_{it}(\delta_i)$ and $\ln L_{it}$ have a unit root. Therefore, applying the Central Limit Theorem:

$$\sqrt{N}(\tilde{\phi} - \phi) \xrightarrow{d} N(0, V) \text{ when } N \rightarrow \infty \text{ and } \frac{\sqrt{N}}{T^\kappa} \rightarrow 0,$$

where

$$V = E(\eta_1 \eta_1').$$

The V matrix can be estimated consistently by:

$$\hat{V} = \frac{1}{N} \sum_{i=1}^N [(\hat{\phi}^{(i)} - \tilde{\phi})(\hat{\phi}^{(i)} - \tilde{\phi})'].$$

It is well known that the endogeneity of the productive factors, which are usually measured with error, and the simultaneity of these variables, are a cause of specification problems in industrial economics models. Under these circumstances, the estimators discussed above are inconsistent, and the coefficients should be estimated by instrumental variables, using as instruments the lagged values of the productive factors. However, given the small size of the time series (15 periods), we will not follow this approach here².

3. DATA DESCRIPTION AND MODEL SPECIFICATION

The data corresponds to the 81 manufacturing sectors of the Encuesta Industrial for the period 78-92, where:

Y_{it} = Gross Value Added in real terms.

N_{it} = Total Number of workers.

H_{it} = Total Hours Worked.

$h_{it} = \frac{H_{it}}{N_{it}}$ = Hours Worked per Worker.

I_{it} = Total Gross Investment in real terms.

K_{i0} = Initial Real Capital Stock³.

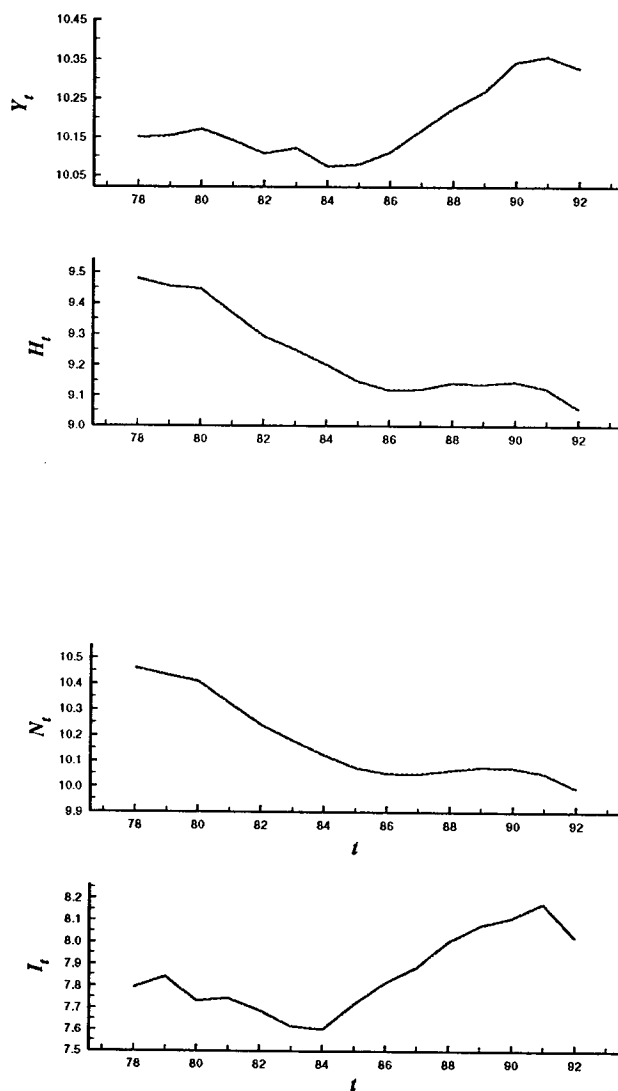
Figure 1 contains the aggregated series for the 81 sectors:

$$Y_t = \frac{1}{N} \sum_{i=1}^N Y_{it}, \quad H_t = \frac{1}{N} \sum_{i=1}^N H_{it}, \quad I_t = \frac{1}{N} \sum_{i=1}^N I_{it},$$

²The results of the instrumental variable estimation are available from the authors upon request.

³Regarding the construction of the investment and capital stock variables see Martín Marcos (1990)

FIGURE 1
Aggregate Series of Y_t , N_t , H_t , I_t .



This figure shows an structural change in the behavior of the series during the

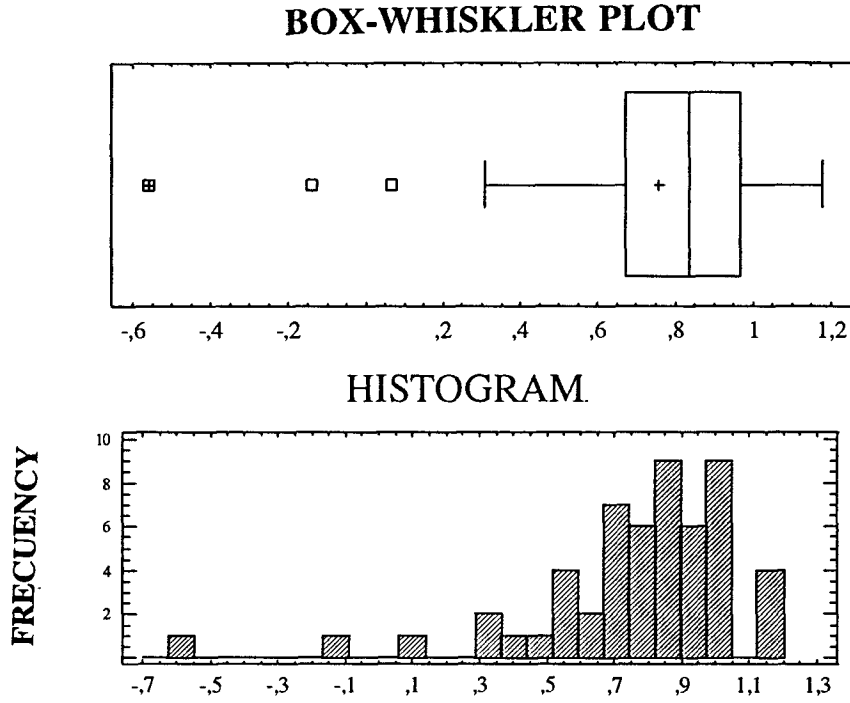
period 85-86. This structural change can be justified in several ways. The dates coincide with the end of the recession in which Spain was immersed since the mid 70's and which affected the industrial sector in a particularly virulent manner. The end of the recession is reflected, on the one hand, in the important change in the entry-exit dynamics of firms in the industry (Segura et al, 1989). And on the other hand, in the change in labor relationships. From 1976 to 1985 the workday was reduced at an annual rate of over 1.5%. In 1985, a workday similar to those of other European countries was reached, and this trend broke down (Jaumandreu, 1987). Finally the 70's and the beginning of the 80's were characterized by a remarkable increase in energy prices which stabilized in 1985. In addition, on January 1, 1986 Spain joined the European Economic Community and VAT was introduced.

The unit root tests cannot reject a unit root in the aggregate series of Y_t , at any reasonable significance level. The tests allow for a constant and a deterministic trend. Tests were also carried out allowing the structural change to take place in 85, and the null hypothesis of the existence of a unit root could not be rejected. Although the lack of stationarity in the aggregate is clear, the behavior of the series in many sectors is stationary.

Figure 2 contains Box and Whisker plots and histogram of the autocorrelation coefficients of Y_{it} for each sector. The sides of the box that appears inside figure 2 correspond to the values of the first and third quartile, the central line correspond to the median, and the + sign corresponds to the mean. The extremes on the horizontal line, correspond to the maximum and minimum value excluding the extreme values. The points marked as \boxplus are three times away the interquartile interval, and the points marked as \boxminus are 1.5 times away.

FIGURE 2

Autocorrelation Coefficients of Y_{it} , $i = 1, \dots, 81$.



The autocorrelation coefficient of, ρ_i , is estimated by:

$$\hat{\rho}_i = \frac{\sum_{t=2}^T (Y_{it} - \bar{Y}_i) (Y_{it-1} - \bar{Y}_i)}{\sum_{t=2}^T (Y_{it} - \bar{Y}_i)^2},$$

where $\bar{Y}_i = T^{-1} \sum_{t=1}^T Y_{it}$. The mean value of $\hat{\rho}_i$ is 0.75592 and the standard deviation is 0.0350; the 95% confidence interval for the mean value of ρ_i is [0.6697, 0.8421] with a median of 0.834; 25 % of the correlation coefficients are greater than 0.95, which is probably why the aggregate series shows a unit root.

We have grouped the sectors in 10 large groups. The correspondence of our classification with the classification used in the national accounts (NACE-CLIO R25) and the E.I. sectors is shown in table 1.

TABLE 1
CLASSIFICATION USED IN THIS STUDY AND ITS RELATION WITH THE
NACE-CLIO R25 CLASSIFICATION AND THE SECTORS OF THE E.I.

GRUPO	NACE-CLIO R25	E.I.
1	2 Metallic Minerals and Steel	9-11
2	3 Minerals and non Metallic Products	12-18
	4 Chemical	19-30
3	5 Metallic Products	31-35
4	6 Machinery	36,37
	7 Office Machinery and others	38,46
5	8 Electrical Material	39,40
	9 Transport Material	41-45
6	10 Food	47-64
7	11 Textil and Shoes	65-74
8	12 Paper	80-82
9	13 Rubber and Plastic	83,84
10	14 Woods, Cork and others	75-79,85-89

To complete our model specification, we include in Z_{it} a dummy variable that reflects a structural change after 1986, and a variable to approximate technological progress. In such a long period of time, technological change must have played an important role. However the introduction of this variable in the production function poses difficulties. If we introduce this variable as a deterministic trend, there is an identification problem between the trend and the parameter of the depreciation rate. An heuristic argument can be developed as follows. The capital equation may be written as:

$$K_{it}(\delta_i) = K_{it-1}(\delta_i)[1 - \delta_i + r_{it}], \quad (13)$$

where $r_{it} = I_{it}/K_{it-1}(\delta_i)$ only depends on δ_i in the long-run. If we approximate $K_{it}(\delta_i)$ in (13) around $r_{it} - \delta_i = 0$, equation (13) can be rewritten as:

$$K_{it}(\delta_i) \approx K_{i0} \exp \left\{ -\delta_i t + \sum_{\tau=1}^t r_{i\tau} \right\}, \quad (14)$$

If $Z_{it} = t$ in (7), we can write

$$\ln Y_{it} \approx \alpha_i + (\gamma_i - \beta_{Ki}\delta_i)t + \beta_{ki} \left\{ \ln K_{i0} + \sum_{\tau=1}^t r_{i\tau} \right\} + \beta_{Li} \ln L_{it} + \varepsilon_{it}.$$

For each value of β_{ki} , $\xi_i = \gamma_i - \beta_{Ki}\delta_i$ is satisfied for infinite values of δ_i and γ_i , for any given ξ_i . γ_i and δ_i are not identified. This explains the extreme values obtained in the estimation of δ_i when a deterministic trend is included in the model. Prucha and Nadiri (1993) and Denia et al (1996) recognize the need for accounting for technical progress but they do not incorporate it in an explicit way in their work, probably because of the identification problem shown above.

In this article we will construct the technical progress series by computing the Solow residual, (Solow,1957)⁴. Starting from an aggregate Cobb-Douglas production function with constant returns to scale,

$$\ln Y_t = a_t + \varphi_t \ln H_t + (1 - \varphi_t) \ln K_t,$$

where $Y_t = \frac{1}{N} \sum_{i=1}^N Y_{it}$, $H_t = \frac{1}{N} \sum_{i=1}^N H_{it}$ and $K_t = \frac{1}{N} \sum_{i=1}^N K_{it}$, the changes in production are given by changes in labor weighted by the share of labor in production and/or changes in capital weighted by the share of capital in production and/ or changes in technical progress,

$$\Delta \ln Y_t = \Delta a_t + \varphi_t \Delta \ln H_t + (1 - \varphi_t) \Delta \ln K_t.$$

Therefore, the changes in technical progress are those changes in production that cannot be explained through changes in the productive factors,

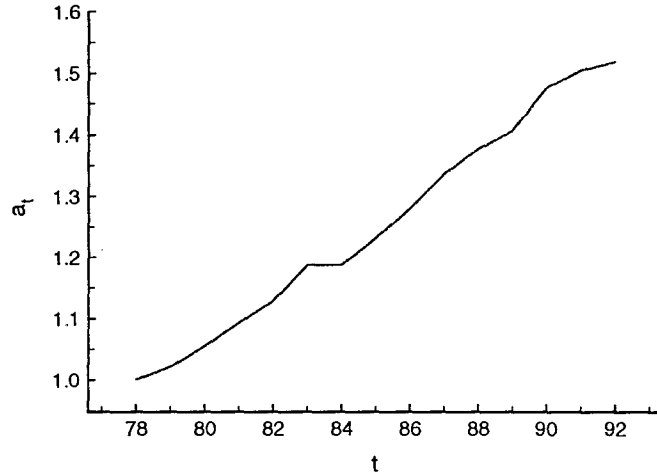
$$\Delta a_t = \Delta \ln Y_t - \varphi_t \Delta \ln H_t - (1 - \varphi_t) \Delta \ln K_t.$$

⁴This procedure has been used by Dolado J.J., J.L. Malo de Molina and A. Zabalza (1986), among others.

Where $\varphi_t = \frac{\varpi_t}{Y_t}$ is the labor share in production, ϖ_t are wages and salaries in real terms, Y_t is real value added, H_t are total hours worked and K_t is the capital stock series in real terms obtained by Martín Marcos (1990). We can obtain the level of a_t , by, $a_t = \Delta a_t + a_{t-1}$, normalizing $a_0 = 1$.

The technical progress series obtained is shown in figure 3.

FIGURE 3
Technical Change



Beyond technical progress, there are other factors which are potentially important when modelling a production function. We have tried to take into account some of these factors in this study.

Two alternative models are proposed :

- Model 1:

- Labor Factor: Total number of hours worked, $L_{it} = H_{it}$.
- Other variables: $Z_{it} = \begin{pmatrix} D_t \\ a_t \end{pmatrix}$, $D_t = \begin{cases} 1 & \text{if } t \leq 1985 \\ 0 & \text{if } t > 1985 \end{cases}$

- Model 2:

- Labor Factor : Total Number of workers, $L_{it} = N_{it}$.
- Other variables: $Z_{it} = \begin{pmatrix} D_t \\ a_t \\ \ln h_{it} \end{pmatrix}$, $D_t = \begin{cases} 1 & \text{if } t \leq 1985 \\ 0 & \text{if } t > 1985 \end{cases}$

In Model 1 the production function is given by:

$$f_i(K_{it}(\delta_i), L_{it}, Z_{it}, \theta^{(i)}) = A_i [K_{it}(\delta_i)]^{\beta_{Ki}} L_{it}^{\beta_{Li}} \exp \{ \gamma_{1i} D_t + \gamma_{2i} a_t \},$$

and taking logs,

$$\ln Y_{it} = \alpha_i + \beta_{Ki} \ln K_{it}(\delta_i) + \beta_{Li} \ln L_{it} + \gamma_{1i} D_t + \gamma_{2i} a_t + \varepsilon_{it}.$$

In Model 2 the production function is given by:

$$f_i(K_{it}(\delta_i), L_{it}, Z_{it}, \theta^{(i)}) = A_i [K_{it}(\delta_i)]^{\beta_{Ki}} L_{it}^{\beta_{Li}} h_{it}^{\beta_{hi}} \exp \{ \gamma_{1i} D_t + \gamma_{2i} a_t \},$$

and taking logs,

$$\ln Y_{it} = \alpha_i + \beta_{Ki} \ln K_{it}(\delta_i) + \beta_{Li} \ln L_{it} + \beta_{hi} \ln h_{it} + \gamma_{1i} D_t + \gamma_{2i} a_t + \varepsilon_{it},$$

where γ_{2i} is the coefficient of technical change, $[K_{it}(\delta_i)]^{\beta_{Ki}} L_{it}^{\beta_{Li}}$ is output per hour worked and $h_{it}^{\beta_{hi}}$ the workday. This model is more flexible than the previous one, since it allows for the utilization of the labor factor to adjust to the fluctuations of production in two ways: the variation in the number of workers, and the variation

in the number of hours worked per worker. As Hamermesh (1993) points out, if the changes in the workday have been large, the introduction of the workday in the model should improve its capacity to capture the changes in the utilization rate of labor. The production function represents the flows of inputs and outputs, and therefore, the capital stock and hours of work data should be transformed in service flows data. The coefficient of the workday is a short-run elasticity which allows the adjustment of the labor factor in face of transitory demand fluctuations. The value of β_{hi} is an empirical matter and will depend on the effect of the changes of the workday on production, for a given technology⁵. To take into account that capital utilization is a decision variable for the firm, we would need an index of productive capacity.

Finally, when estimating a production function we have to bear in mind that the quantities of capital and labor used, the level of production and the price level are generated by a set of simultaneous relations, and therefore neither capital nor labor can be treated as exogenous variables. However, in the short-run, the firm's productive capacity may be taken as given, and therefore its capital stock as well. But the endogeneity of the labor factor will have to be taken into account through appropriate econometric techniques.

4. EMPIRICAL RESULTS IN RANDOM COEFFICIENTS MODELS

In this section we discuss the estimation of models 1 and 2 by the random coefficients procedure using the 81 manufacturing sector of the E.I.

Table 2 presents the estimated values of the components of ϕ , confidence intervals and inference tests using all the estimated parameters for the 81 sectors⁶.

The first thing to notice from the table, is that the values of the means and medians of the coefficients are reasonable. The elasticity of capital (10%) is slightly below the share of capital in production, and the elasticity of labor is slightly above labor's share in production (86%) and, when we test for constant returns to scale using the

⁵See Jaumandreu (1987)

⁶An asterisk denotes that the coefficient is significant at the 5% level.

means of the estimated parameters, the hypothesis cannot be rejected ($t = -0.29$) at the 5% significance level, suggesting that the model is well specified.

TABLE 2
NLLS estimation by random coefficients of model 1

	$\hat{\delta}_i$	$\hat{\alpha}_i$	$\hat{\beta}_{Ki}$	$\hat{\beta}_{Li}$	$\hat{\gamma}_{1i}$	$\hat{\gamma}_{2i}$
Mean	.0889	-.5175	.1049	.8643	.0045	.5611
Median	.0347	-.9262	.1270	.8195	.0137	.5891
Standar Deviation	.2818	9.604	.7150	.5405	.2063	2.33
t est. (mean)	2.840*	-.4849	1.321	14.39*	.1997	2.161*
Rank signed test (median)	2.523*	1.134	2.179*	7.603*	.7015	2.650*
Minimum	-.9451	-40.12	-2.652	-.6310	-.9334	-7.466
Maximum	.8994	41.23	2.139	2.100	.6433	7.408

The mean value of the depreciation rate is 8.9%. This value is greater than that estimated by Denia et al (1996), (5.5 to 6%), using aggregated data for the Spanish economy, or that obtained by Nadiri and Prucha (1993), (5%), for the American manufacturing sector. Although both, the data and the sample periods are different, these estimations could serve as a benchmark. It is important to emphasize, that the time period used in this study covers, and is almost restricted to, the Spanish economic recession that lasted from the mid 70's to the mid 80's and which was particularly strong in the industrial sector. Therefore, it is reasonable to obtain a higher depreciation rate in this period. The mean is very sensitive to extreme values, so the median can be used as an additional source of information. The median of $\hat{\delta}_i$ is 3.47%, a very small value although more similar to that obtained by other authors. The difference between the mean and the median shows that there exists a large number of outliers. As the equation has been estimated in levels, the coefficient $\hat{\gamma}_{2i}$ cannot be directly interpreted as the rate of change of technical progress. This will

be the result of multiplying $\hat{\gamma}_{2i}$ by the rate of change of a_t . This coefficient has the correct sign and magnitude and is significant. The structural change dummy variable takes on a very small value and it cannot be rejected at the 5% confidence level that its true value is zero. The constant term has a much higher variability than the other coefficients but it cannot be rejected at any reasonable level of significance, that its true value is zero. These last results are not surprising if we take into account that we are estimating 6 parameters with 15 observations in a highly non-linear context.

Table 3 contains the depreciation rates for each of the ten groups which are obtained in this random coefficients model.

TABLE 3

NLLS Estimation by random coefficients of δ for each of the 10 groups in model 1

GROUP	$\hat{\delta}$
1. Metallic Minerals and Steel (3)	0.007
2. Minerals and non Met. Prod. + Chemical (19)	0.090
3. Metallic Products (5)	0.128
4. Machinery + Off. Machin. and others (4)	0.018
5. Electrical Mat. + Transport Mat. (7)	0.167
6. Food (18)	0.133
7. Textil and Shoes (10)	0.036
8. Paper (3)	0.160
9. Rubber and Plastics (2)	0.015
10. Woods, Cork and others (10)	0.029

Although the number of sectors in each group is small, many of these depreciation rates are reasonable. The more extreme values correspond to the groups that have the smallest number of sectors. These depreciation rates are in general higher than

those obtained with conventional methods, since this methodology takes account not only physical decay, but also of obsolescence⁷.

The estimation results of model 2 appear in the following table. With this new specification the mean value of the depreciation rate is 9.8% while the median is only 2.7%. The capital and labor elasticities are slightly higher than those estimated above. When we test for the constant returns to scale hypothesis, the value of the t statistic is 0.15, so that the null hypothesis cannot be rejected. The coefficient of the workday is 59.8% and significant which implies that there are costs associated with increasing the workday beyond its standard level. Technical progress has a mean value of 39% and a median of 26%, and we cannot reject that the coefficient is equal to zero at the 5% significance level. The structural change dummy variable appears to be zero.

TABLE 4
NLLS Estimation by random coefficients of model 2

	$\hat{\delta}_i$	$\hat{\alpha}_i$	$\hat{\beta}_{Ki}$	$\hat{\beta}_{Li}$	$\hat{\beta}_{hi}$	$\hat{\gamma}_{1i}$	$\hat{\gamma}_{2i}$
Mean	.0981	-.559	.1457	.8741	.5987	-.008	.3950
Median	.0279	-.393	.1542	.8008	.5681	-.004	.2619
Estandar Deviation	.3161	11.67	.7460	.7020	1.370	.1911	2.112
t Estatistic (mean)	2.79*	-.431	1.757	11.20*	3.931*	-.396	1.683
Rank signed test(median)	2.36*	.776	3.192*	7.25*	3.818*	.056	1.953
Minimum	-.878	-51.12	-3.41	-1.44	-3.32	-.797	-7.50
Maximum	1.01	55.93	2.22	3.055	4.02	.393	7.41

In general, model 2 behaves slightly worse than model 1; it has a larger variance, the number of sectors with coefficients that take on extreme values is larger, and some of the depreciation rates obtained, depart from the levels that we would have

⁷See Risueño (1997).

expected ex ante. These results are probably due to the fact that this model has an additional parameter and the time series is very short.

Table 5 contains the depreciation rates estimated with model 2 for the ten groups. The estimated parameters using this model have a larger variance than those of model 1. We obtain a negative depreciation rate for group 1, and a depreciation rate above 34% for group 8.

TABLE 5

NLLS Estimation by random coefficients of δ for the 10 groups using model 2

GROUP	$\hat{\delta}$
1.Metalic Min. and Steel. (3)	-0.048
2.Minerals and non Met. prod. + Chemical (19).	0.155
3. Metalic Products (5)	0.081
4. Machinery + Office Mach. and others.(4)	0.191
5. Electrical Mat. .+ Transport Mat. (7)	0.126
6. Food (18)	0.061
7. Textil and Shoes (10)	0.062
8. Paper (3)	0.341
9. Rubber and Plastics (2)	0.030
10 Woods, Cork and others (10)	0.027

When we compare these estimation results with those of model 1, we observe large discrepancies in almost all groups. The most important are those for group 4, whose depreciation rate was 1.8%, in model 1 and is 19%, in model 2 and for group 8, whose depreciation rate changes from 16 to 34%. The groups that obtain similar depreciation rates in both models are group 10, whose depreciation rate changes from 2.9 to 2.7%, and group 9, whose depreciation rate changes from 1.5 to 3%.

After examining the results obtained with the two models using the random coefficients procedure, we can reach the following conclusions:

i) With this procedure we get capital and labor elasticities which are approximately equal to the factors' shares in production, (φ_t) , and constant returns to scale as predicted by economic theory.

ii) The workday coefficient has a value of 55 to 60%, which implies that there are costs associated with increasing the workday beyond its standard length.

iii) The aggregate depreciation rate is about 8.9 to 9.8%. These values are reasonable although higher than those obtained by other authors with other samples.

iv) The coefficients of technical change have the appropriate sign and magnitude. The coefficient is larger and more significant when estimated by model 1. This suggests a negative correlation between technical change and the workday.

v) The structural change dummy variable, and the constant term, are not significant, and the latter has a very large variance.

vi) Model 1 seems more stable than model 2. Given the short time series, this is probably due to the additional parameter in model 2.

These results suggest that both the model specifications and the estimation procedure are sound. However, when evaluating the results, we must consider the following:

i) The time series is very short relative to the number of parameters to be estimated. As Pesaran & Smith (1995), recognize, when T is small the estimated values of the coefficients are very imprecise, in particular if the explanatory variables cannot be considered strictly exogenous. This lack of degrees of freedom problem exacerbates in the presence of specification problems, or if the model is nonlinear.

ii) In this framework, instrumental variables estimation is difficult to implement to solve the endogeneity problem. The errors are autocorrelated and lagged values of the explanatory variables are not, in general, good instruments. Even if we had a different set of variables that could be used as instruments, it would be almost impossible to find a unique set of instruments that could be used for all sectors.

These considerations may help explain some of the results obtained and, in particular, the large disparities observed in the depreciation rates of the ten groups.

Therefore, the use of the mean value of the estimated parameters of each equa-

tion has its weakness. We have a very short time series relative to the number of parameters to be estimated. This gives rise to objective functions that behave non-cooperatively and the estimators tend to be unstable. We observe that the distribution functions of the estimated parameters have very thick tails and many outliers. These considerations suggest that an analysis with pooled data, assuming fixed coefficients, should be carried out. This procedure gives rise to consistent estimators when the series are stationary and the non-linear parameter is fixed. In all the estimations we have carried out, the variances of δ_i are smaller than those of the rest of the parameters. There are obvious advantages in using all the data simultaneously. The estimators are more stable and the estimations are less affected by outliers. This is why we present in the next section the estimation results using pooled data.

5. EMPIRICAL RESULTS WITH POOLED DATA

In this section we will first present the results of estimating an aggregate δ for models 1 and 2 using pooled data for the 81 manufacturing sectors of the E. I. In the tables heteroskedasticity robust t-statics are shown in brackets and heteroskedasticity and autocorrelation robust t-statistics are shown in squared brackets.⁸

As can be seen from table 6, the results are different from those obtained with the random coefficients procedure: The capital elasticity is greater and the labor elasticity is smaller than those obtained in the previous section (actually, they are almost equal). These coefficients, however, are similar to those reported by Denia et al (1996) using aggregated series for the Spanish economy.

⁸As has been discussed above, the errors of the model are heteroscedastic and autocorrelated. Therefore the variance covariance matrices are estimated robustly to the presence of heteroskedasticity and/ or autocorrelation.

TABLE 6

Pooled data estimation of models 1 and 2 for an aggregate δ^9

COEF	MOD 1	MOD 2
δ	.1107 (4.944) [2.401]	.0958 (4.401) [2.146]
α	.4733 (2.015) [1.120]	.0460 (.1905) [.0994]
β_K	.4741 (39.10) [14.59]	.4741 (39.52) [14.83]
β_L	.4508 (30.19) [10.21]	.4613 (31.12) [10.64]
β_h		.8043 (13.87) [4.739]
γ_1	.0418 (.8774) [.7905]	.0381 (.8291) [.7653]
γ_2	-.0120 (-.0682) [-.0404]	-.0335 (-.1909) [-.1122]
SSR	113.868	108.910
SER	.3068	.3002
\bar{R}^2	.9340	.9368

⁹t -statistics robust to heteroskedasticity (•) and to heteroskedasticity and autocorrelation [•]

The workday elasticity is greater than that obtained in the random coefficients procedure, but it still implies costs associated to the adjustment of the workhours. The coefficient of technical progress does not have the right sign, or size in any of the two models. The structural change dummy variable is positive, but is not significant.

When the constant returns to scale hypothesis is tested, it is rejected for model 1, the t-statistic being $t = -3.072$. However the hypothesis cannot be rejected for model 2, the t-statistic being $t = 1.668$. These results suggest that model 2 is better specified than model 1.

These regressions were also run with only those sectors whose residual autocorrelation coefficients were less than .8 and, therefore, the series involved were not expected to be $I(1)$. The results were not substantially different from those obtained for the whole sample. For both specification the first order autocorrelation coefficient of the errors was estimated. In all cases it was approximately 90%. Such high autocorrelation may be signalling that the models are badly specified due to the heterogeneity of the productive sectors being considered. Therefore, these regression were also estimated including dummy variables in the constant term to allow for heterogeneity among the sectors.

Whenever dummy variables were included in the constant term, we obtained a negative depreciation rate; however, the capital elasticity diminished and the labor elasticity increased taking values similar to those obtained in the random coefficients models, while the correlation coefficients diminished sharply, although they did not disappear. We also tried to correct for autocorrelation adding to the model the lagged values of the dependent and explanatory variables without success. This is not surprising giving the results in Pesaran & Smith (1995) and because the production function represent a theoretical relation in which dynamics are difficult to justify. The models were also estimated using the within estimator. With this specification we obtained negative depreciations rates, a labor coefficient next to 1 and a capital coefficient next to zero and not significant, which suggest that the within estimator

is not appropriate to capture the long-run relation of the variables, and that the endogeneity of the explanatory variables, which are necessarily measured with error, exacerbates by taking first differences, since the measurement error variance increases.

To test the robustness of the results obtained using pooled data estimation, we estimated models 1 and 2 using a capital stock series constructed from the E.I.¹⁰. These estimations support those obtained in the non-linear case. The estimated coefficients using this series are almost identical to the ones obtained when the depreciation rate is estimated jointly with the parameters of the production function. The linear model was also used to test the Cobb-Douglas specification against the CES and Translog, the results being favorable to the Cobb-Douglals.

In the following table we present the estimation results of models 1 and 2 when we allow for 10 depreciation rates, one for each of the large groups.

¹⁰This series is CAPIR81. See Martín Marcos (1990).

TABLE 7

Pooled data estimation of models 1 and 2 for 10 different δ 's¹¹

	MOD 1			MOD 2		
	COEF	Est t (H)	Est t (H y A)	COEF	Est t (H)	Est t (H y A)
α	.3834	1.693	.9475	.0729	.3064	.1593
δ_1	.0663	2.226	1.086	.0551	1.926	.9378
δ_2	.0664	2.363	1.034	.0682	2.465	1.088
δ_3	.1776	6.139	2.519	.1531	5.598	2.302
δ_4	.0564	1.787	.6321	.0561	1.811	.6447
δ_5	.1099	3.149	1.434	.0853	2.648	1.235
δ_6	.0743	2.976	1.312	.0662	2.747	1.235
δ_7	.1705	6.955	2.988	.1419	6.061	2.660
δ_8	.0706	2.803	1.461	.0560	2.303	1.200
δ_9	.0927	4.185	2.137	.0712	3.382	1.741
δ_{10}	.1790	5.958	2.372	.1438	5.207	2.063
β_K	.4203	28.94	11.52	.4325	30.31	12.21
β_L	.5038	29.71	10.47	.4996	30.02	10.67
β_h				.7803	13.68	4.517
γ_1	.0361	.7775	.7342	.0320	.7065	.6808
γ_2	.0878	.5285	.3153	.0375	.2248	.1353
SSR	108536			105.548		
SER	.3007			.2966		
\bar{R}^2	.9366			.9383		

When we allow for 10 different depreciation rates, the estimation results improve. In all cases considered, the capital elasticity diminishes and the labor elasticity in-

¹¹Est t (H):heteroskedasticity robust t statistics

Est t (H y A): heteroskedasticity and autocorrelation robust t statistics

creases to about 42% and 50% respectively. The hours of work elasticity is slightly smaller than that obtained when we considered an aggregated depreciation rate, although it is still larger than that obtained using the random coefficients procedure. Technical progress has the correct sign, although it is very small and not significant and the structural change dummy variable is not significant. All the estimated depreciations rates are plausible. They are all positive and below 20% and, unlike those obtained with the random coefficients procedure, they are robust to model specification. When the constant returns to scale hypothesis is tested it is rejected for model 1 ($t = -3.024$). However, the hypothesis cannot be rejected for model 2 ($t = 1.226$).

As in the case in which we considered an aggregate depreciation rate, the correlation coefficients are very high, and when dummy variables are included in the constant term to allow for heterogeneity, the depreciation rates become negative. If we impose the restriction of positive depreciation rates, the coefficients are almost zero, not significant and the NLLS algorithm does not converge.

6. COMPARISON BETWEEN THE DEPRECIATION RATES AND CAPITAL STOCKS OBTAINED IN THIS STUDY WITH THOSE AVAILABLE FOR THE SAMPLE.

The depreciation rates obtained in this study are larger than those obtained using conventional methods for their calculation and, therefore, give rise to smaller capital stocks. In the following table we present the capital-output ratios for the aggregate capital stocks obtained from the estimation of models 1 and 2 using the random coefficients procedure (c.v.), the pooled data procedure (d.a.) and those of the CAPIR81 series.

If we observe the capital-output ratios obtained in this study, we can see that all the capital stocks are similar. They reflect a destruction of installed capital equipment until 88, then the stocks stabilize, and increase systematically from then on. When analyzing the capital stocks calculated using the depreciations rates estimated in the previous sections, it must be taken into account that the initial capital stock has not

been estimated consistently, as another parameter of the production function, since it cannot be identified¹². Therefore, we cannot make flat assertions about the moment in which the decline of the capital stock stabilizes or its initial magnitude. The capital-output ratios of these series are clearly smaller than those of the CAPIR81 series. The latter fluctuate around 1.48% until the year 88, and then they increase continuously as well.

TABLE 8

Capital-output ratios obtained in this study by random coefficients estimation (c.v.) and pooled data estimation (d.a.) and those obtained from the series CAPIR81

AÑO	RCPM1(c.v.)	RCPM2(c.v.)	RCPM1(d.a.)	RCPM2(d.a.)	RCPCAPIR81
78	1.480	1.480	1.480	1.480	1.480
79	1.439	1.425	1.407	1.429	1.482
80	1.390	1.364	1.330	1.370	1.473
81	1.346	1.310	1.263	1.319	1.467
82	1.330	1.285	1.226	1.296	1.484
83	1.282	1.230	1.161	1.243	1.467
84	1.241	1.182	1.105	1.196	1.453
85	1.212	1.147	1.064	1.163	1.449
86	1.198	1.128	1.040	1.145	1.457
87	1.185	1.111	1.019	1.129	1.466
88	1.183	1.106	1.009	1.125	1.483
89	1.195	1.114	1.014	1.133	1.513
90	1.210	1.126	1.023	1.146	1.546
91	1.245	1.159	1.053	1.180	1.599
92	1.266	1.176	1.068	1.198	1.637

¹²For an explanation of why the initial capital stock is not identify see Mauleón(1997).

In the following table we compare the depreciation rates obtained for the 10 groups using models 1 and 2 estimated by random coefficients and pooled data and the depreciation rates implicit in the CAPIR81 series.

TABLE 9

Comparison of the depreciation rates obtained with models 1 and 2 using the two estimation procedures and those obtained using a conventional method.

	MOD 1 (c.v.)	MOD 2 (c.v.)	MOD 1 (d. a.)	MOD 2 (d.a.)	CAPIR81
δ_1	.007	-.048	.0663	.0551	.0522
δ_2	.090 (.110)	.155 (.126)	.0664	.0682	.0622
δ_3	.128	.081	.1776	.1531	.0665
δ_4	.018 (-.020)	.191	.0564	.0561	.0604
δ_5	.167 (.169)	.126 (.047)	.1099	.0853	.0597
δ_6	.133 (.141)	.061 (-.085)	.0743	.0662	.0475
δ_7	.036 (.145)	.062 (.077)	.1705	.1419	.0626
δ_8	.160	.341 (.154)	.0706	.0560	.0532
δ_9	.015	.030	.0927	.0712	.0665
δ_{10}	.029	.027	.1790	.1438	.0465

As can be seen from the table, with the pooled data procedure (d.a.) we get very similar depreciation rates for both models and, if we compare them with the depreciation rates implicit in the capital stock series calculated by Martín Marcos (1990)¹³, we can see many similarities although there are also important differences. In general, the depreciation rates obtained in this article are larger than those calculated using a traditional method. The major agreements occur when we compare the latter with those obtained with model 2, in particular for groups 1, 2, 4, 8 and may be 9, and

¹³These series were constructed using the Perpetual Inventory Method approximating the depreciation rate by $\delta = \frac{1}{T}$, where T is the useful life of the asset.

perhaps 6, while there are clear differences for groups 3, 7, 10 and to a lesser extent for group 5.

If it is true that the groups for which we find the greatest differences are those which are built with the smallest number of sectors, or those that, as sector 10, are composed of very different sectors, and therefore their estimations are less reliable, it is also true that conventional methods do not take into account the depreciation due to obsolescence. We should find high depreciation rates in those groups that have experienced major firm exit rates in any of its sectors, or important technological changes. Group 7, Textile, suffered a major crisis during the whole sample period, so its reasonable to find a high depreciation rate for this group. The same could be said of group 3, which suffered an important fall in production during the period 80-85, and of the Transport Material component of group 5, which had an important decline during the period 82 - 87, and of groups 9 and 10, whose production decreased markedly during the years 80-85 and 81-86, respectively. Group 1 did also experience production declines similar to some of the ones mentioned before, but in the latter case, the decline did not happen in a systematic way. For the whole sample, this group experienced recession spells that alternated with production recoveries.

Given the small amount of data that we have, the random coefficients procedure (c.v.) does not seem appropriate to calculate disaggregated depreciation rates. There are large discrepancies between the depreciation rates obtained with the two models. In general, the least credible depreciation rates correspond to the groups with the smallest number of sectors to carry out their estimation, but we observe instability in all cases. Model 1 seems slightly more stable than model 2, and some of the groups that obtained the highest depreciation rates in the pooled data models also get them with this model and procedure, in particular groups 3, 5 and, may be, 7. Beyond this, it is difficult to find any other similarities.

7. CONCLUSIONS

In this article we have estimated the depreciation rate of the manufacturing sectors jointly with the parameters of a production function, using alternative methods of estimation and different specifications of the model within the Cobb-Douglas technology.

To carry out the estimation we have a very short time series relative to the number of parameters to be estimated. The time series of the different sectors behave in a very heterogeneous manner. We observe stationary and non-stationary series. There are large differences among the estimated coefficients of the different sectors. The explanatory variables cannot be taken as strictly exogenous, the errors are autocorrelated and the model is non-linear.

Given the characteristics of the data, there is not a completely satisfactory econometric procedure to carry out inferences about the parameters. If we use pooled data, the variability of the parameters may result in inconsistent estimations. This problem exacerbates if the series are integrated. If we use the random coefficients procedure we can obtain very imprecise estimates, given the short time series and the non-linearity of the model.

In this study we use both estimation procedures and compare them. The results obtained with both methods are reasonable. The estimated aggregate depreciation rate and the capital and labor elasticities are within the bounds of the values we would have expected a priori. With the random coefficients procedure we get an aggregate depreciation rate of 8.9 to 9.8%, capital and labor elasticities that approximately correspond to the shares of the factors in production, and we cannot reject constant returns to scale in the aggregate; The coefficient of the workday indicates that there are costs associated with increasing the workday beyond its standard length. However, the estimated depreciation rates tend to be unstable and very sensitive to outliers, as can be seen from the comparison of the means and medians of the estimated parameters. When we tried to obtain disaggregated depreciation rates for groups

of sectors, they were very unstable. With the pooled data procedure we obtained aggregate depreciation rates from 9.6 to 11%, a capital elasticity which is higher, and a labor elasticity which is lower, than the shares of these factors in production, and we reject the constant returns to scale hypothesis for model 1. The coefficients of the workday, technical progress and the structural change dummy variable are stable and, in general, have the correct sign and size. The depreciation rates obtained with this procedure are very robust to changes in model specification and changes in sample size. With this estimation procedure it is possible to obtain group-disaggregated depreciation rates which are stable. The results of the pooled data procedure show that estimations improve when we take into account the fluctuations in the workday as approximation to changes in the utilization rates of productive factors. With this specification we cannot reject constant returns to scale, the coefficient of the workday is always significant, and the \bar{R}^2 of the regressions of model 2 are always higher than those of model 1.

The capital stocks obtained by both estimation procedures are very similar and show a huge destruction of capital that took place during most of the sample period, as a consequence of the industrial crisis. The capital-output ratios implied by these series are next to unity, and are clearly smaller than those obtained with a capital stock calculated by conventional methods.

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